AC VOLTAGE RATIO MEASUREMENT

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WHILE RESISTANCE, mass, length, etc., are measured in terms of standards kept by national laboratories, voltage ratio, being dimensionless, has no national, legal standard. Thus anyone may make ratio measurements and may claim any accuracy he feels he can justify, even if it exceeds that claimed by a national laboratory.

At reasonable frequencies and voltages (e.g., 1 kHz, 100 v), available ratio-measuring systems have sensitivities on the order of one part-per-billion (ppb) of input; comparisons on voltage dividers can be repeated to this accuracy on successive measurements. With two dividers (one reversed with respect to the other), the correction for the 0.5 point can be repeatedly determined to within a few ppb.

Measurement Methods

One way to evaluate the accuracy of voltage ratio measurements is to compare results obtained using several different techniques. Many methods have been used for this measurement. Several use precision resistors, but at 100 v and 1 kHz, low value resistors dissipate excessive power and therefore change, while high value resistors exhibit excessive phase shift. Thus these methods were avoided.

Three alternative methods were selected: the *cyclic capacitor* method of Cutkosky and Shields (Ref. 1), the *bootstrap* method of Sze (Ref. 2) and Hill (Ref. 3), and a *straddling* method based on the common reversing technique.

Cyclic Capacitor Method

In this technique, a decade transformer is compared with several capacitive dividers (each capacitor is used in both halves of the divider and the average of all measurements is taken). The beauty of the method lies in its mathematics. For the 3:1 divider (Fig. 1), each capacitor is connected serially to the high input and the others to the low. The average of the three ratios is given in Eq. 1. For accuracy, the capacitors must maintain their values during the measurement cycle; they must have low temperature and voltage coefficients and the measurements should be made in a well controlled room.

$$\frac{1}{3}\left(\frac{C_1}{C_1+C_2+C_3}+\frac{C_2}{C_1+C_2+C_3}+\frac{C_3}{C_1+C_2+C_3}\right)=\frac{1}{3} \quad (1)$$

The circuit for these measurements (Fig. 2), contains three-terminal capacitors that have appreciable capacitance from each terminal to the case which is tied directly to the output of the divider under test. This connection causes negligible loading error because the output impedance is low, the stray capacitances form a divider of almost the same ratio, and the cyclic rotation of the capacitors also rotates these strays such that this source of error is averaged out.

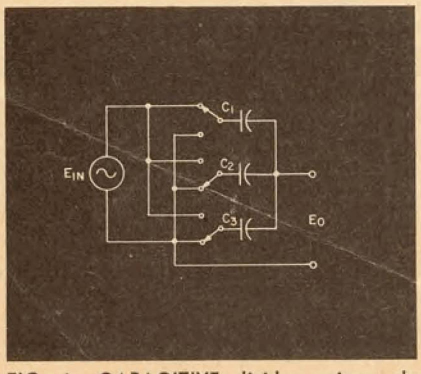


FIG. 1. CAPACITIVE divider, using only three capacitors

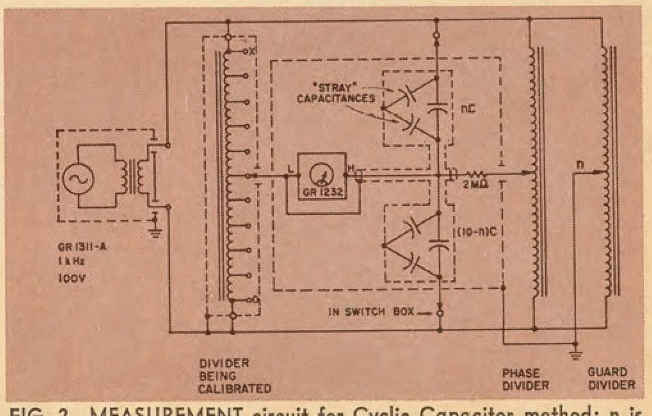
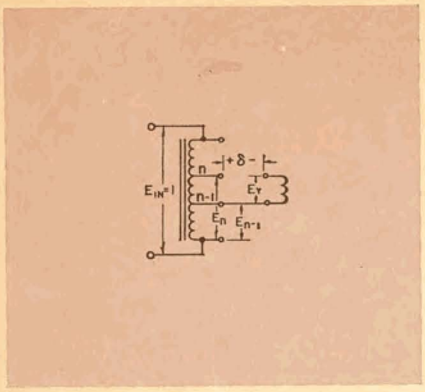


FIG. 2. MEASUREMENT circuit for Cyclic Capacitor method; n is the number of tenths of input



◀ FIG. 3. USE of a tapped transformer for the bootstrap method

The Bootstrap Method

This method uses a shielded 10:1 transformer (Fig. 3) as a yardstick. Each one-tenth step measurement may be inaccurate by an amount δ (Eq. 2), but the sum of the steps must be unity. This latter condition (Eq. 3) permits all voltages to be determined by solving to obtain the transformer voltage E_y, as in Eq. 4. To evaluate each step, sum all voltages from zero, as in Eq. 5.

$$E_n - E_{n-1} = \delta + E_y \quad (2)$$

$$10E_y + \sum_{j=1}^{10} \delta_j = 1 \quad (3)$$

$$E_y = \frac{1}{10}\left(1 - \sum_{j=1}^{10} \delta_j\right) \quad (4)$$

$$E_n = \sum_{k=1}^{n} \delta_k + nE_y \quad (5)$$

In actual dividers, the difference between the zero and full-scale voltages is not equal to the input voltage, because of the voltage drop in the wiring. The input voltage must be measured at the high and low terminals if Eq. 3 is to be valid. The corrections for the 0 and 10 positions can be determined from the difference between the output at these settings and the values at the input terminals.

While the voltage, E_y, can be any value, it is desirable that it be close to $E_{in}/10$. The error in E_y will then be small, so that fractional changes in this error will not be important. E_y must be constant, independent of ratio setting; this requires no capacitance to either winding from variable voltages, and suggests a doubly shielded transformer.

A two-stage transformer (Ref. 3,4) reduces the error due to voltage drop in the primary by sampling the flux and adding additional voltage. The full input voltage (Fig. 4) is applied to one winding. Because of the voltage drop due to z_1, e_2 and e_3 are low. The dif-

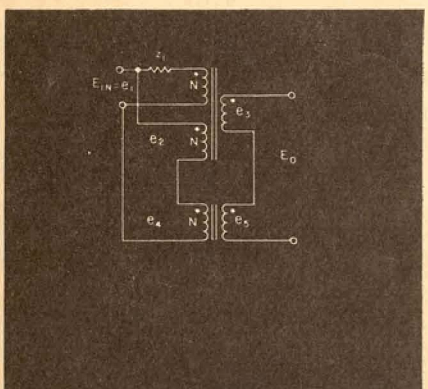


FIG. 4. TWO-STAGE transformer, used to reduce error due to losses

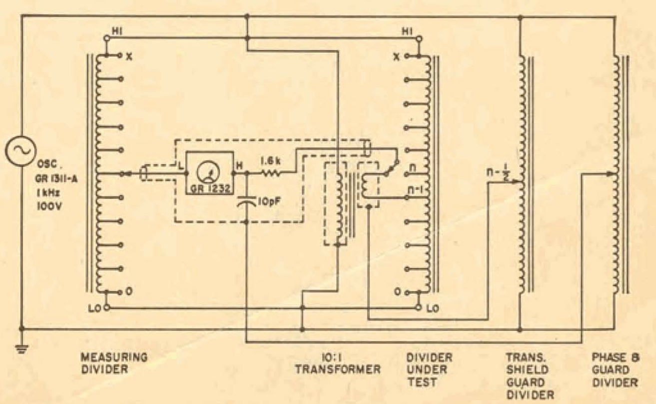
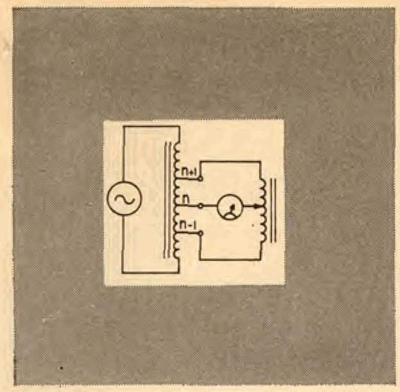


FIG. 5. CONNECTION diagram for bootstrap method; n is the number of tenths of input



◀ FIG. 6. DECADE transformer straddling a pair of 1/10 steps

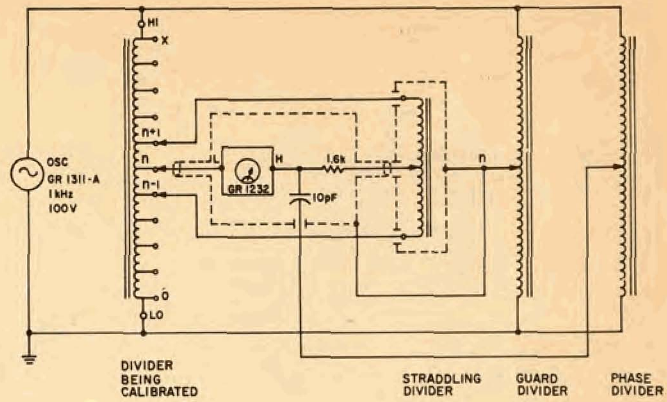


FIG. 7. CONNECTION circuit for straddling method

ference between e_{in} and e_2 is applied to a second transformer whose output is added to e_3 as a correction. Such two-stage transformers with errors of less than 20 ppb of input are not difficult to find. With the bootstrap method (Fig. 5), measurements and calculations for all 1/10 steps can be made in one hour.

The Straddling Method

The correction for the 0.5 point can be found by reversal of one divider with respect to another. Continuing such division gives ¼ points, ⅛ points, etc., (Ref. 5). Unfortunately, most dividers are decade rather than binary, so this subdivision is not suitable. With a known ratio between pairs of adjacent decade steps, however, it is possible to calculate the correction for each step. These ratios can be determined with a second decade transformer straddling each pair of 1/10 steps (Fig. 6).

The arithmetic required to evaluate each point after measurement is conveniently performed with a computer. The divider must have all 1/10 taps brought out (Fig. 7). The correction for the straddling divider at the 0.5 point is determined by lead reversal. The straddling divider loads the divider under test, but in practice, the resulting error is almost negligible. The magnitude of the change caused by loading can be measured by placing a third divider in parallel with the straddling unit, and the resulting change of setting used as a correction. The high and low input terminals are used for the measurement so that the total voltage is unity by definition.

Results

Measured corrections made by two runs using each method showed a maximum spread of points less than 30 ppb. Multiple runs, made by the bootstrap method, all fell within this spread, as did the 0.5 point determined by simple reversal.

An NBS calibration, which states ±200 ppb accuracy, differed by a maximum of 50 ppb from the measured average. The repeatability of the results described here suggests that the average of the measurements is within 20 ppb of true value.

One note of caution for others attempting these measurements. The dividers must be carefully demagnetized before measurement. Previous overloads (removed at peak current) caused magnetization that resulted in changes in calibration of up to 10 ppb. Such overloads may occur in a comparison circuit if the two dividers are set to widely differing values and the detector circuit has low impedance. Cyclic demagnetization restores the original calibration.

References

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